INVESTIGATION OF THE OCCURRENCE OF CONVECTION IN HORIZONTAL LIQUID LAYERS

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The occurrence of convection in horizontal layers of ethanol, distilled water, and ethylene glycol heated from below was investigated by the interferometric method. The limits of roller-type convection and its effect on the magnitude of heat flux through the layer were determined.

The interferometric method described in [1] for investigating thermal processes in liquids with the use of the IAB-451 instrument as a twin-wave diffraction interferometer and a laser as the light source was used by us to determine the limits of occurrence of convection in horizontal liquid layers enclosed between two rigid surfaces and heated from below. The experimental device, experimental method, and some preliminary results were described in [2-4]. Therefore, we will present here the results of generalizing the data on determination of the limits of occurrence of convection and experiments on measurement of the heat fluxes through the layer. In passing we will discuss the problem of the possibility of occurrence of convection in our experiments due to nonisothermicity of the surfaces bounding the layer, which was mentioned in the article of A. V. Lykov and B. M. Berkovskii [5].

We will recall that the experimental cell that enclosed the investigated layer consisted of two hollow brass plates, two plane-parallel glass plates, and two Teflon spacers. Water from two thermostats was passed through the brass plates and each was heated to the required temperature. Determination of the transition from the heat conduction regime to the convection regime and the calculation of ΔT corresponding to this transition were done from the interferograms of the temperature field in the layer. Figure 1 shows

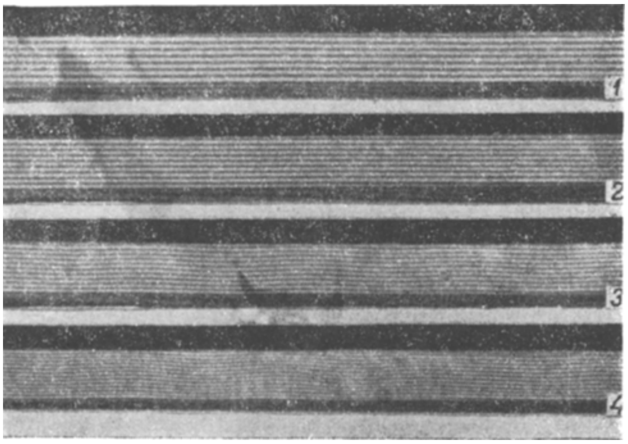

Fig. 1. Interferograms of temperature field in layer of distilled water (d = 2.90 mm): 1) ΔT = 1.93°, Ra = 580, conduction; 2) ΔT = 2.34°, Ra = 710, conduction; 3) ΔT = 2.57°, Ra = 750, convection; 4) ΔT = 3.31°, Ra = 1000, convection.

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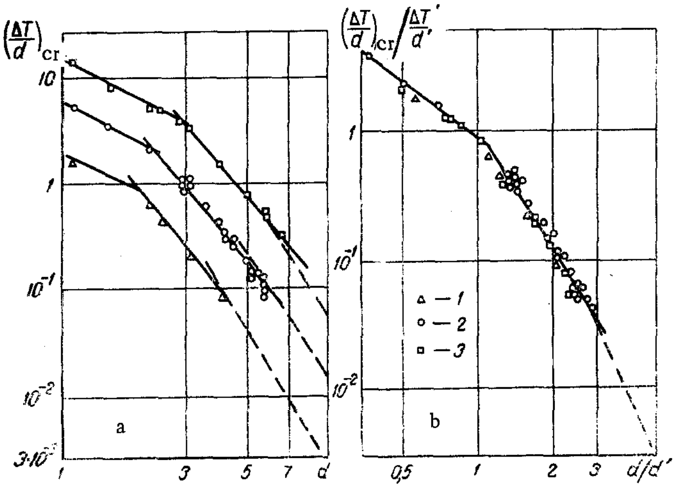

Fig. 2. Critical value of the temperature gradient $(\Delta T/d)_{cr}$, deg/mm, vs layer thickness d, mm (a) and generalization of this dependence (b): 1) ethanol; 2) distilled water; 3) ethylene glycol.

the interferograms of the temperature field corresponding to the conduction and convection regimes. On the basis of the appearance of the interferograms we can state that we observed not a three-dimensional hexagonal, but a two-dimensional roller structure of convection, the possibility of existence of which was shown in theoretical [6, 7] and experimental [8, 9] studies, but the conditions of occurrence of which have not been investigated thoroughly.

A determination of the limits of occurrence of convection in horizontal layers of ethanol, distilled water, and ethylene glycol showed that disturbance of the conduction regime occurs at Ra_{cr} less than 1700 [3, 4], whereby the smaller the thickness of the layer, the lower the values of Ra_{cr} at which this disturbance occurs. The occurrence of instability in the layer, recorded from the interference pattern of the temperature field, is not characterized by any particular value and depends both on the layer thickness and on the properties of the liquid. Moreover, layers of the same thickness filled with the same liquid had different values of Ra_{cr} depending on the average temperature T_{av} at which the experiment was performed. We note that T_{av} of the layer was selected equal to room temperature in order to minimize possible heat losses through the vertical boundaries of the layer.

Experiments showed that each liquid has its own maximum layer thickness at which convection has a roller structure. For layers of greater thickness the character of the interferograms changed markedly, indicating a three-dimensional structure of convection (apparently hexagonal cells). From these interferograms it was impossible to calculate the temperature difference at the layer boundaries. Therefore, we investigated layers whose maximum thickness for ethanol was 3.10 mm, for water 5.80 mm, and for ethylene glycol 6.80 mm.

The critical value of the temperature gradient is plotted against the layer thickness d in a logarithmic coordinate system in Fig. 2a. The figure shows that the experimental points lie nicely on broken straight lines whose corresponding sections for the three investigated substances are parallel to one another. The dashed lines in Fig. 2a were plotted on the basis of the calculated points for layers whose thickness exceeded the thickness of our investigated layers and the occurrence of convection in which was assumed to be at $Ra_{cr} \approx 1700$.

The data obtained (Fig. 2a) show that all three broken straight lines can be superposed if the measurement results are presented in a system of dimensionless coordinates with characteristic thicknesses and gradients corresponding to the break of the straight lines plotted for the experimental points. The result of such generalization is shown in Fig. 2b, where we can note three rectilinear sections depending on the layer thickness: a section characterized by the occurrence of cellular convection when $d/d' \geq 3$ and two sections characterized by the occurrence of roller-type convection for layers of comparatively small thickness when $d/d' \leq 1$ and $d/d' = 1$-3. On the whole this broken line separates the conduction regime in the layer (below the broken line) from the region of the convection regime (above it).

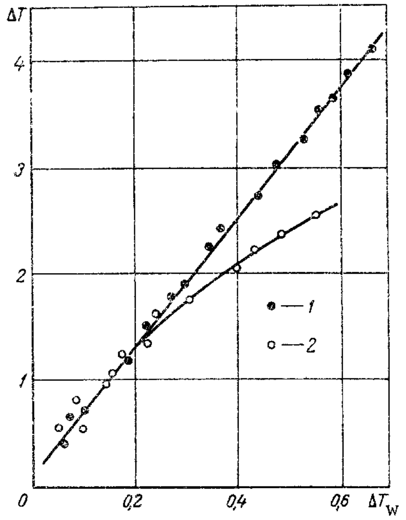

Fig. 3

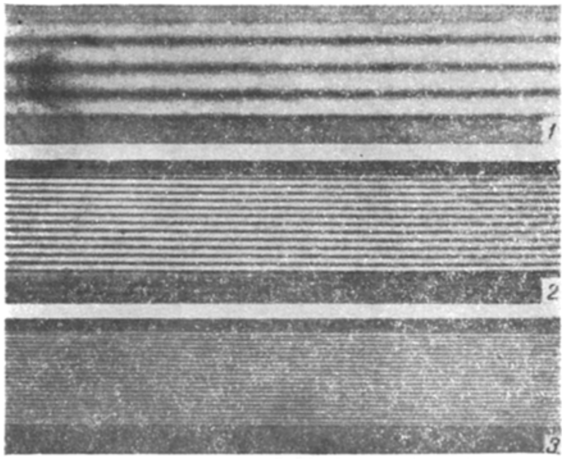

Fig. 4

Fig. 3. Relation between the temperature difference in the layer and temperature difference in the cell wall: 1) heating from above; 2) heating from below.

Fig. 4. Interferograms of temperature field in horizontal layer of distilled water with heating from above (d = 5.28 mm): 1) ΔT = 0.89°, Ra = 1640; 2) ΔT = 3.15°, Ra = 5790; 3) ΔT = 5.63°, Ra = 10,350.

Taking into account that the ratio of the length of the layer horizontally to its thickness varied in our experiments from 4.5 to 27, we can assert that the results obtained are applicable not just to our experimental cell.

We can make the following assumption concerning the occurrence of convection at values of $\mathrm{Ra}_{\mathrm{cr}}$ less than the value $\mathrm{Ra}_{\mathrm{cr}} \approx 1700$ usually taken for a layer enclosed between two rigid boundaries.

In analytic solutions of the problem of instability of a horizontal liquid layer heated from below, the layer was regarded as infinitely long horizontally. Therefore, the boundary conditions were assigned only on the horizontal boundaries. In our case, as in all experimental investigations of a horizontal layer, vertical walls are necessarily present, the boundary conditions on which should undoubtedly be taken into account. It is rather difficult to assign these conditions correctly, but it is characteristic that their consideration even in a knowingly simplified form [10] leads to a change of the stability limit.

Quite interesting is the problem of the effect of roller-type convection on the heat flux through the layer. To elucidate this problem we fabricated a new experimental cell of steel 1Kh18N9T whose heat conductivity was considerably less than that of brass. It differed from the previous cell also in that the wall of one of the plates was 20 mm thick and 1-mm-diam. holes were made in it at a distance of 15 mm for the junctions of differential thermocouples.

The heat flux through the liquid layer and through the cell wall is written respectively so:

$$q = \lambda_{\mathrm{l}}^{\mathrm{eq}} \frac{\Delta T}{d}, \tag{1}$$

$$q = \lambda_{\mathrm{s}} \frac{\Delta T_{\mathrm{w}}}{d_{\mathrm{w}}}. \tag{1a}$$

Neglecting the heat losses from the vertical walls of the cell, we can write

$$\lambda_{\mathrm{l}}^{\mathrm{eq}} \frac{\Delta T}{d} = \lambda_{\mathrm{s}} \frac{\Delta T_{\mathrm{w}}}{d_{\mathrm{w}}}. \tag{2}$$

The temperature difference at the boundaries of the liquid layer was measured from the number of interference fringes, and the temperature drop in the cell wall ΔT_{w} by a differential thermocouple.

TABLE 1. Data on Measurement of Heat Fluxes in Horizontal Liquid Layers

No.	Substance	d, mm	ΔT, °C	ΔT_W, °C	$\frac{\Delta T}{d}$, $\frac{°C}{mm}$	T_{av} °C	ε_c	Ra
1	H_2O	1,50	2,50	0,97	1,32	18,5	1,00	92
2	The same	1,50	2,90	1,18	1,93	19,0	1,00	112
3	" "	1,50	3,50	1,60	2,34	19,0	1,00	126
4	" "	1,50	4,07	1,80	2,72	19,5	1,00	163
5	" "	1,50	5,35	2,50	3,57	20,1	1,05	215
6	" "	1,50	5,55	2,66	3,70	20,2	1,07	233
7	" "	1,50	5,75	2,80	3,84	20,3	1,09	242
8	" "	1,50	6,00	2,98	4,00	20,5	1,10	256
9	$C_2H_4(OH)_2$	4,30	0,95	0,13	0,22	19,5	1,00	240
10	The same	4,30	2,00	0,27	0,47	20,0	1,00	520
11	" "	4,30	2,60	0,37	0,60	20,5	1,00	680
12	" "	4,30	3,30	0,44	0,77	21,0	1,00	890
13	" "	4,30	4,60	0,65	1,07	21,5	1,00	1300
14	" "	4,30	4,80	0,70	1,12	21,7	1,06	1370
15	" "	4,30	4,90	0,79	1,14	21,9	1,12	1430
16	" "	4,30	5,00	0,84	1,16	22,3	1,13	1500
17	" "	4,30	5,15	0,93	1,20	22,5	1,14	1560

Note. 1-4, 9-13) Conduction regime; 5-8, 14-17) convection regime.

Equation (2) can be represented in the form

$$\Delta T = \frac{\lambda_s}{\lambda_l^{eq}} \cdot \frac{d}{d_w} \Delta T_w \tag{3}$$

The values of λ_s and d_w remain constant in all experiments, d is assigned after assembling the cell, and $\lambda_l^{eq} = \lambda_l$ remains constant as long as a conduction regime exists in the layer. Thus, if we plot the experimental points in coordinate system $\Delta T - \Delta T_W$, under conditions of a thermal regime they will lie on a straight line whose slope is determined by the numerical value of the complex $(\lambda_s \lambda_l^{eq}) \cdot (d/d_w)$. In the case of the occurrence of convection in the layer the value of λ_l^{eq} increases in comparison with λ_l as convection develops, as a consequence of which the numerical value of the coefficient of ΔT_w in (3) begins to change and the points on the graph deviate from a straight line.

Therefore, each experiment was carried out both with heating of the layer from below and with heating from above. We see in Fig. 3 that the experimental points obtained on heating the layer from above, i. e., under conditions of a conduction regime in the layer, lie nicely on a straight line. On heating the layer from below the points lie on the same straight line as long as a conduction regime exists in the layer. Then as convection occurs and develops an ever greater deviation of the experimental points from a straight line is observed.

The intensity of the convection process in a layer is often characterized by the convection coefficient [11]

$$\varepsilon_c = \lambda_l^{eq} / \lambda_l \tag{4}$$

which is equal to unity under conduction regime conditions and increases as convection develops.

Table 1 presents some data on measurement of heat fluxes by the method indicated above in horizontal layer of distilled water and ethylene glycol.*

As indicated, a series of experiments was carried out with heating of the layer from above, i. e., when conditions for the occurrence of convection are absent, which was actually confirmed by the interferograms of the temperature field in the layer and by thermocouple measurements. Here we will dwell in slightly more detail on this problem in connection with the following circumstance.

The possibility of occurrence of convection in a horizontal cavity filled with gas or liquid with heating from above was shown in [5]. For this purpose it is sufficient to make the temperature of the upper face either different over the surface or variable in time. It was indicated that convection can occur at arbitrarily small deviations from isothermicity and steadiness. The authors explain by means of this effect the fact that in a number of experimental works, including in ours [3], the occurrence of convection in a horizontal layer was noted at values of Ra_{cr} less than the usually accepted value $Ra_{cr} \approx 1700$.

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*T. V. Gurenkov participated in conducting the experiments.

Without disputing the authors' conclusions concerning the possibility of occurrence of convection under the conditions presented above, we will show that in our experiments the cause of convection could not have been deviation from isothermicity and stability, the proof of which are the results of experiments with heating of the layer from above.

It is known that on heating a horizontal layer from above the heat is transmitted through the layer by conduction no matter how great the temperature difference between the boundaries of the layer. It is understandable that in a real case the side walls can begin to have an effect under certain conditions and convection can occur in some part of the layer even with such a direction of the temperature gradient. However, the interferograms of the temperature field in the layer (with heating from above) obtained by us (Fig. 4) showed a high degree of isothermicity of the surfaces enclosing the layer, although the temperature difference at the boundaries reached values corresponding to $Ra \approx 10^4$.

The interference fringes, which can be identified with isotherms, represent straight lines parallel to the layer boundaries. This indicates a conduction regime in the layer. The degree of isothermicity of the layer boundaries can be characterized quantitatively by the fact that displacement of the interference fringe at some place of the layer from its original position by a distance equal to the width of the fringe (this displacement can easily be noted visually) corresponds, for our experimental conditions, to a change of temperature: 0.04° for ethanol, 0.25° for water, and 0.08° for ethylene glycol. However, such deviations were not noted on a single interferogram of the temperature field in the case of heating from above.

The design of the IAB-51 instrument allows not only photographing the investigated object but also continuous observation of it. Observation of the interference pattern of the temperature field in the layer after establishing the necessary temperature difference at the boundaries showed the absence of any noticeable changes of the temperature field with the course of time.

It is clear that such a degree of isothermicity of the layer boundaries and steadiness of the temperature field is retained also on heating the layer from below, since in this case all experimental conditions remained as before and only the direction of the temperature gradient changed. Therefore, the disturbance of the conduction regime in horizontal liquid layers occurring on heating from below, which was noted in our experiments, occurs by no means because of nonisothermicity of the layer boundaries or departure from steadiness.

Thus we can note that by means of the interference method it was possible to record the occurrence of convection in horizontal liquid layers, to determine the structure of the convection flows, and to calculate ΔT_{cr} corresponding to the occurrence of convection. Additional experiments showed that the observed roller-type convection intensifies heat transfer in the layer.

NOTATION

ΔT_{cr}	is the critical value of the temperature difference corresponding to the occurrence of convection in the layer;
Ra_{cr}	is the critical value of the Rayleigh number;
T_{av}	is the average temperature of the liquid layer;
d	is the thickness of the liquid layer;
$(\Delta T/d)_{cr}$	is the critical value of the temperature gradient corresponding to the occurrence of convection in the layer;
$\Delta T'/d'$	is the value of the temperature gradient corresponding to the break of the straight lines plotted from the experimental points;
q	is the heat flux;
d_w	is the distance in cell wall at which the thermocouple junctions are located;
ΔT_w	is the temperature drop in the cell wall;
λ_l	is the coefficient of heat conductivity of the liquid;
λ_l^{eq}	is the equivalent value of the coefficient of heat conductivity of the liquid;
λ_s	is the coefficient of heat conductivity of steel;
ε_c	is the convection coefficient.

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